JSAEM Studies in Applied Electromagnetics and Mechanics, 11

APPLICATIONS OF ELECTROMAGNETIC PHENOMENA IN ELECTRICAL AND MECHANICAL SYSTEMS

Proceedings of
The First Japanese-Australian Joint Seminar on Applications of Electromagnetic Phenomena in Electrical and Mechanical Systems
16-17 March 2000, Adelaide, Australia

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Applications of Electromagnetic Phenomena in Electrical and Mechanical Systems
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Data based least squares for the currents evaluation

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ABSTRACT: Modern electronic devices are always composed of the printed circuit boards (PCBs) with multiple layers. If the currents on the PCBs can be visualized without disassembling the electronic devices, the testing and inspection of the devices can be carried out in a most efficient manner. This paper proposes a methodology for estimating the current distribution on the PCB with multiple layers. The estimation of the current distributions on a PCB from locally measured magnetic fields can be always reduced into solution of an ill-posed inverse problem. This paper reveals that the conventional least squares method gives a reasonable solution of our inverse approach. Thus, we succeeded in realizing a highly reliable non-destructive testing methodology for electronic devices.

1 INTRODUCTION

Modern electronic devices are always composed of the printed circuit boards (PCBs). Current visualization on the PCBs is of paramount information for the maintenance, inspection and electromagnetic compatibility engineering of the modern electronic devices. To evaluate the current distribution on the PCBs without decomposing or destroying the devices, it is necessary to solve an inverse problem [1-4].

This paper proposes a new methodology to visualize the current distribution on the multi-layered PCB from the locally measured magnetic fields by means of the data based least squares method [3,5]. Previously, we have proposed a loop current model in order to construct the database current distributions [3]. This method has a versatile capability to evaluate any types of the current distributions on the single PCB. However, this model may be difficult to apply an evaluation of the currents on the PCB with multiple layers. In order to apply our least squares method to the evaluation of the current distributions on the PCB with multiple layers, in the present paper, we employ the typical current distribution model as well as the loop current model for database constructing. The PCB is generally composed of a copper sheet on the bottom playing the role of the ground. The loop current model represents the current distribution on the earthed copper sheet, and also the other typical current distributing models correspond to the currents in the particular electronic circuit elements. Further, we have to measure the magnetic field distributions on the front- as well as at the backside of the PCB with multiple layers.

Thus, we have applied our data based least squares method to the evaluation of the current distributions on the PCB with multiple layers. An initial test suggests that our approach makes it possible to evaluate the current distributions in the PCB with multiple layers.

2 VISUALIZATION OF THE CURRENTS

2.1 Input vector

Let us consider a magnetic field measured at the both front and backsides of a PCB with n by n resolution:

$$H_{n\times n} \in f_x(x_i, y_j), f_y(x_i, y_j), f_z(x_i, y_j) \qquad (1)$$
$$i = 1,2,\cdots n, \quad j = 1,2,\cdots n,$$

where the functions f_x, f_y, f_z refer to the $x-$, $y-$ and $z-$ components of the magnetic fields; x_i, y_j denote the $i-th$ on x-axis and $j-th$ on y-axis locations of a measured surface, respectively.

Arranging the components of magnetic fields $H_{n\times n}$ into a column-wise form gives input vectors $\mathbf{Y}$ with $3\times n\times n$-th order as

$$\begin{aligned}\mathbf{Y} = [&f_x(x_1,y_1), f_x(x_2,y_1),.., f_x(x_n,y_1), f_x(x_1,y_2), f_x(x_2,y_2),..,\\ &f_x(x_n,y_2),.., f_x(x_{n-1},y_n), f_x(x_n,y_n),\\ &f_y(x_1,y_1), f_y(x_2,y_1),.., f_y(x_n,y_1), f_y(x_1,y_2), f_y(x_2,y_2),..,\\ &f_y(x_n,y_2),.., f_y(x_{n-1},y_n), f_y(x_n,y_n),\\ &f_z(x_1,y_1), f_z(x_2,y_1),.., f_z(x_n,y_1), f_z(x_1,y_2), f_z(x_2,y_2),..,\\ &f_z(x_n,y_2),.., f_z(x_{n-1},y_n), f_z(x_n,y_n)]^T.\end{aligned} \quad (2)$$

2.2 System matrix

The currents visualization in single-layered PCB requires using only the loop currents model. But, for the PCB with multiple layers, it was difficult to use only such a model as the database and to visualize it. So, the well-known typical models in the front surface are used as the database, too.

2.2.1 System matrix of loop currents model

Let us assume the $m\times m-th$ unit loop currents on backside of PCBs:

$$C_{L_{m\times m}}^{(k)} \in u(x_i, y_j),$$
$$i = 1,2,.., m, \qquad j = 1,2,.., m, \qquad k = 1,2,.., m\times m, \quad (3)$$

where functions $u(x_i, y_j)$ takes 1 at the position (x_i, y_j) on the backside surfaces of PCBs.

By means of the unit loop current model, the magnetic fields with n by n resolution are represented by

$$D_{L_{n\times n}}^{(k)} \in G_x^{(k)}(x_i, y_j), G_y^{(k)}(x_i, y_j), G_z^{(k)}(x_i, y_j),$$
$$i = 1,2,.., n, \qquad j = 1,2,.., n, \qquad k = 1,2,.., m\times m, \quad (4)$$

where $G_x^{(k)}(x_i, y_j), G_y^{(k)}(x_i, y_j), G_z^{(k)}(x_i, y_j)$ are the Green's functions transferring the effects of the $k-th$ unit current to the measured position (x_i, y_j) in terms of the x-, y- or z-component of the magnetic fields, respectively [3]. Thereby, $k-th$ column vector of a system matrix is given by

$$\begin{aligned}\mathbf{d}_L^{(k)} = [&G_x(x_1,y_1), G_x(x_2,y_1),.., G_x(x_n,y_1), G_x(x_1,y_2), G_x(x_2,y_2),..,\\ &G_x(x_n,y_2),.., G_x(x_{n-1},y_n), G_x(x_n,y_n),\\ &G_y(x_1,y_1), G_y(x_2,y_1),.., G_y(x_n,y_1), G_y(x_1,y_2), G_y(x_2,y_2),..,\\ &G_y(x_n,y_2),.., G_y(x_{n-1},y_n), G_y(x_n,y_n),\\ &G_z(x_1,y_1), G_z(x_2,y_1),.., G_z(x_n,y_1), G_z(x_1,y_2), G_z(x_2,y_2),..,\\ &G_z(x_n,y_2),.., G_z(x_{n-1},y_n), G_z(x_n,y_n)]^T.\end{aligned} \quad (5)$$

Thus, for the loop current model, a system matrix with $3\times n\times n$ -th rows and $m\times m$ -th columns is given by

$$D_L = \left[\mathbf{d}_L^{(1)}, \mathbf{d}_L^{(2)},.., \mathbf{d}_L^{(m\times m)}\right]. \quad (6)$$

2.2.2 System matrix of typical currents model

Let's us consider the typical currents models, whose size and position are different to each other with $p-th$ m by m resolution.

$$C_{T_{m\times m}}^{(l)} = I^{(l)}(x_i, y_j) \quad (7)$$

By means of typical currents database, the magnetic fields with n by n resolution are represented by

$$D_{T_{n\times n}}^{(l)} \in G_x^{(l)}(x_i, y_j), G_y^{(l)}(x_i, y_j), G_z^{(l)}(x_i, y_j),$$
$$i = 1,2,.., n, \qquad j = 1,2,.., n, \qquad l = 1,2,.., p, \quad (8)$$

where $G_x^{(l)}(x_i, y_j), G_y^{(l)}(x_i, y_j), G_z^{(l)}(x_i, y_j)$ are the Green's functions transferring the effects of $l-th$ currents distributions to the magnetic fields at a measured surface. Thereby, $l-th$ column vector of a system matrix is given by

$$\begin{aligned}\mathbf{d}_T^{(l)} = [&G_x(x_1,y_1), G_x(x_2,y_1),.., G_x(x_n,y_1), G_x(x_1,y_2), G_x(x_2,y_2),..,\\ &G_x(x_n,y_2),.., G_x(x_{n-1},y_n), G_x(x_n,y_n),\\ &G_y(x_1,y_1), G_y(x_2,y_1),.., G_y(x_n,y_1), G_y(x_1,y_2), G_y(x_2,y_2),..,\\ &G_y(x_n,y_2),.., G_y(x_{n-1},y_n), G_y(x_n,y_n),\\ &G_z(x_1,y_1), G_z(x_2,y_1),.., G_z(x_n,y_1), G_z(x_1,y_2), G_z(x_2,y_2),..,\\ &G_z(x_n,y_2),.., G_z(x_{n-1},y_n), G_z(x_n,y_n)]^T.\end{aligned} \quad (9)$$

Thus by employing the typical current distribution models, a system matrix with $3 \times n \times n$-*th* rows and p-*th* columns is given by

$$D_T = \left[\mathbf{d}_T^{(1)}, \mathbf{d}_T^{(2)}, .., \mathbf{d}_T^{(p)}\right]. \tag{10}$$

2.3 System of equations

Let Y_{Front} and $Y_{Backside}$ be the magnetic field vectors measured at the front- and backsides of PCB, respectively. Denoting a solution vectors X_{Front} and $X_{Backside}$ with $m \times m$, a system of equation taking into account the both front and backsides magnetic fields is written by

$$\begin{pmatrix} Y_{Front} \\ Y_{Backside} \end{pmatrix} = \begin{pmatrix} D_{T,\ Front} & D_{L,\ Front} \\ D_{T,\ Backside} & D_{L,\ Backside} \end{pmatrix} \begin{pmatrix} X_{Front} \\ X_{Backside} \end{pmatrix}, \tag{11}$$

where D_{Front} and $D_{Backside}$ represent system matrices caused by the magnetic fields measured at the front and backsides of the PCB, respectively. Eq.(11) is formally rewritten in the form of

$$Y = DX, \tag{12}$$

In most cases, a number of equations $2 \times (3 \times n \times n)$ is much larger than those unknowns $m \times m + p$, so that it is possible to apply a conventional least squares method mean as

$$\mathbf{X} = \left[D^T D\right]^{-1} D^T \mathbf{Y}. \tag{13}$$

2.4 Visualization of the currents on the PCBs

By considering the equation, it is revealed that the elements in the solution vector X_{Front} are corresponding to the weights w_{T_i} $(i = 1, 2, \cdots p)$ to the typical currents distributions. This means that the visualized front currents V_{Front} are given by

$$V_{FRONT_{m \times m}} = \sum_{i=1}^{p} w_{T_i} C_{T_{m \times m}}^{(i)}. \tag{14}$$

Also, the elements of solution vector $X_{Backside}$ are corresponding to the weights $w_{L_j} (j = 1, 2, \cdots m \times m)$ to each of the unit loop current distributions. The currents distribution on the backside of the PCB is obtained by

$$V_{BACKSIDE_{m \times m}} = \sum_{j=1}^{p} w_{L_j} C_{L_{m \times m}}^{(j)}. \tag{15}$$

3 EXPERIMENTAL VERIFICATION

To verify our methodology for the PCB with multiple layers, we have carried out an intensive experiment. We have used an experiment model shown in Fig.1. The black points represent the measured points at the top and bottom surfaces.

3.1 Magnetic fields by unit loop currents

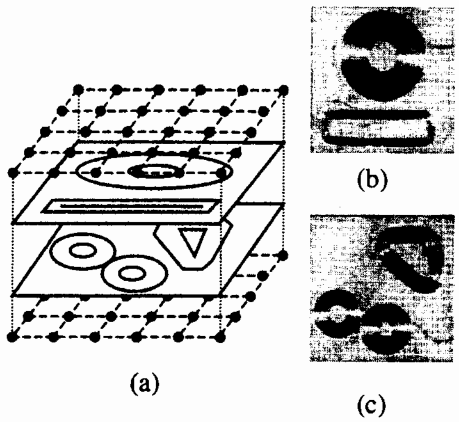


Figure 1 (a) Two-layered experimental model; (b) front- and (c) backside exciting coils of the model.

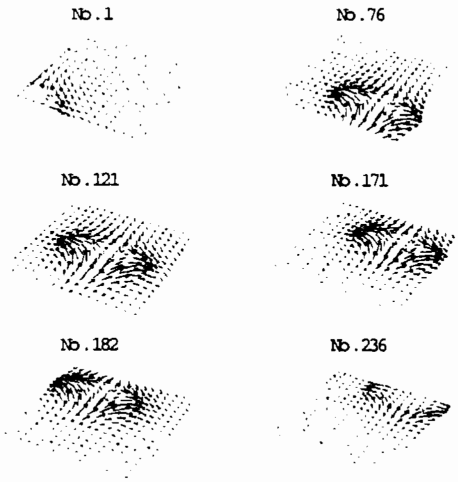
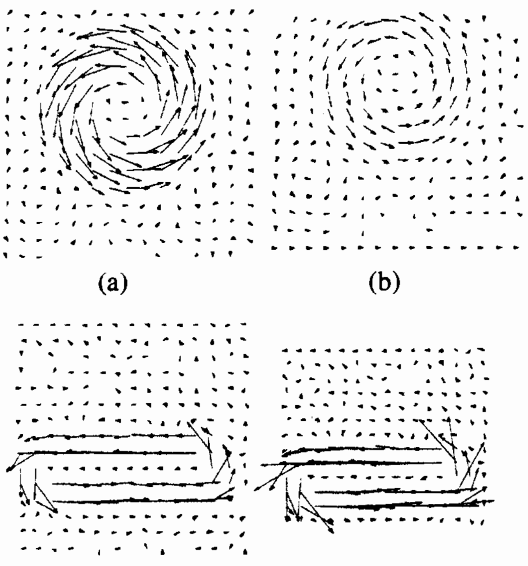


Figure 2 Sample examples of the magnetic fields caused by the unit loop currents.

Fig.2 shows the sample examples of the magnetic fields $D^{(k)}_{n\times n}, k = 1,76,121,171,182,236$ caused by the unit loop currents $C^{(k)}_{m\times m}, k = 1,76,121,171,182,236$, on a backside copper sheet of PCB. In Fig.2, we set the parameters $m = n = 16$, so that our problem is reduced into computing the $m\times m = 256$ loop currents from the $3\times n\times n = 768$ magnetic fields.

3.2 Magnetic fields by typical currents

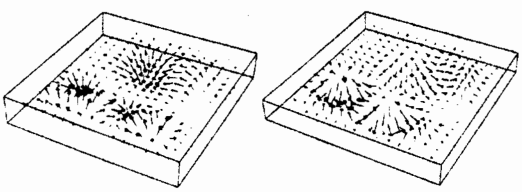
Fig.3 shows the sample examples of the magnetic fields caused by the known model elements on a surface of PCB. The current and position in each of the elements take the distinct values.



(a) (b)

(c) (d)

Figure 3 The current databases taking the (a)-(b) circular and (c)-(d) square path.



(a) (b)

Figure 4 Measured magnetic fields at the (a) front- and (b) backside of PCB.

3.3 Measured magnetic fields

Fig.4 shows the measured magnetic fields at the front- and backsides of the exciting coils.

3.4 Least squares solution

By means of Eq.(13), the solution vector $X_{Backside}$ is obtained as shown in Fig.5. The elements of the solution vector $X_{Backside}$ shown in Fig.5 become the weights of the loop currents in Eq.(3).

3.5 Visualization of the currents

By means of Eq.(15), we have computed the current distributions as the weighted sum of the typical models as well as loop currents. Fig.6 shows the obtained current distributions together with the coil frames.

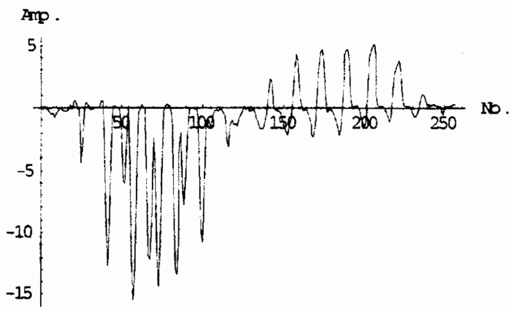
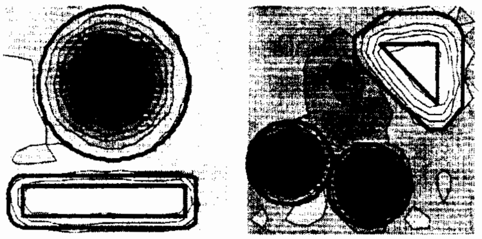


Figure 5 The elements of the solution vector $\mathbf{X}_{Backside}$.

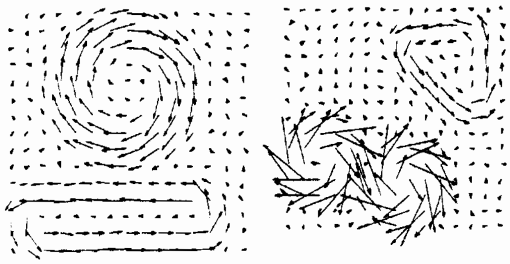
Fig.7 shows the current vector distribution obtained by taking a rotation operation of the computed currents distribution in Fig.6. By observing the currents vectors in Fig.7, there is a small difference in the position between exact and computed vectors. Also, noise which is an effect caused by front surface currents, may be found. However, major computed vectors are well corresponding to the exact ones.

Fig.8 shows the measured and computed magnetic field distributions at the front surface of PCB. From the results shown in Fig.8, it is obvious that the least squares method gives good solutions. By employing the typical models, it is possible to visualize the current distributions on the two-layered test model.



(a) (b)

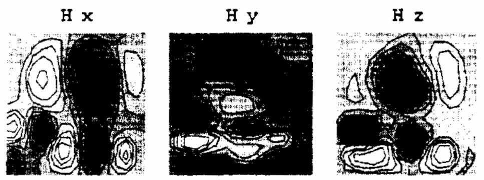
Figure 6 (a) Front- and (b) backsides computed loop circuit distributions with the coil frames (bold black lines).



(a) (b)

Figure 7 (a) Front- and (b) backsides computed currents vector distributions.

(a) X-Comp. (b) Y-Comp. (c)Z-Comp.



(d) X-Comp. (e) Y-Comp. (f) Z-Comp.

Figure 8 (a)-(c) Computed and (d)-(f) measured magnetic field distributions. The X-,Y- and Z-Comp. refer to the x, y and z components, respectively.

4 CONCLUSIONS

As shown above, we have proposed a new inverse approach to visualizing the current distribution on the PCB with multiple layers by measuring the local magnetic fields at the front and backsides of the PCB. Even though the backside currents on the PCB affect the magnetic field distribution at the front side, by means of the databases stored experimentally, our approach has let to a successful result.

Thereby, a condition for which the number of unknown is less than the number of equations has led to use the least squares method. As a result, it has been suggested that the non-destructive testing and inspection of the electronic devices having the PCB with multiple layers, can be carried out in a quite efficient manner.

5 REFERENCES

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