A parameter representing eddy current loss of soft magnetic materials and its constitutive equation

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Previously, Sakaki has proposed a parameter which is closely related to eddy current loss of soft magnetic materials. Also, Saito has proposed a similar parameter to represent the magnetization characteristics exhibiting hysteresis loops. In this paper, we show that these independently proposed parameters are essentially the same ones, and lead to a constitutive equation corresponding to a Chua-type magnetization model. Furthermore, a relationship between our proposed parameter and domain walls is examined.

I. INTRODUCTION

Recently, considerable interest has been focused on the magnetization model for numerically evaluating the hysteretic field in magnetic devices. The model representing magnetization characteristics may be roughly classified into two major categories. One is the Preisach-type model, which assumes that each of the domains has a rectangular hysteresis loop, and the interaction between domains can be introduced by examining local fields acting on domains.[1] Mayergoyz has mathematically generalized this Preisach-type model and shown its versatilities.[2,3] The other is the Chua-type model, which has been primarily worked out using a lumped circuit model for nonlinear inductors exhibiting hysteresis loops. This Chua-type model has been generalized and successfully applied to the calculations of magnetic field, skin depth and loss.[4,5]

Sakaki has proposed a parameter to represent the eddy current loss of soft magnetic materials.[6–8] Also, he has clarified that his parameter depends on the number of domain walls and on material resistivity.

In this paper, we show that one of the parameters of the Chua-type model corresponds to Sakaki's parameter. As a result, a physical interpretation of the Chua-type model is given by means of domain theory.

II. MAGNETIZATION CHARACTERISTICS

A. Models

A specific Chua-type model is given by

$$H+\left(\frac{\mu_r}{s}\right)\left(\frac{dH}{dt}\right)=\left(\frac{1}{\mu}\right)B+\left(\frac{1}{s}\right)\left(\frac{dB}{dt}\right), \tag{1}$$

where H and B are the field intensity and flux density, μ and μ_r are the permeability and reversible permeability, and s is a parameter called hysteresis coefficient, respectively.[4,5]

When we define the reversing and applying point field intensities, H_n, H_p, as shown in Fig. 1, then a Preisach's function can be represented by[9]

$$\psi=\frac{\partial^2 B}{\partial H_p \partial H_n}. \tag{2}$$

According to the Refs. 4 and 5, the Preisach's function ψ in (2) is related with the hysteresis coefficient s in (1) by

$$s=\psi\left(\frac{\partial H}{\partial t}\right). \tag{3}$$

From (3), a Chua-type model (1) can be modified as

$$H+\left(\frac{\mu_r}{\psi}\right)=\left(\frac{1}{\mu}\right)B+\left(\frac{1}{\psi}\right)\frac{\partial B}{\partial H}. \tag{4}$$

In the weakly magnetized region, it is possible to assume that the parameters μ_r, μ, ψ are constant. Then, an initial magnetization curve by (4) can be obtained as

$$\begin{aligned} B &= \mu H + (\mu/\psi)(\mu_r - \mu)[1-\exp(-\psi H/\mu)] \\ &\simeq \mu_r H + (1/2)\psi H^2, \end{aligned} \tag{5}$$

where $\mu \gg \mu_r$ and $\exp(-\psi H/\mu) \simeq 1-(\psi H/\mu)+(1/2)(\psi H/\mu)^2$ are assumed. Equation (5) represents a Rayleigh's initial magnetization curve.[10] Thus, it is obvious that the Preisach-type model is a generalization of Rayleigh's rule covering a highly magnetized region. Furthermore, as an essential feature, it can be considered that there is no frequency dependence on the Rayleigh or Preisach loops. Such a frequency-independent hysteresis loop is sometimes called the dc or static loop.

A frequency-independent term in the Chua-type model (1) is the second term on the left; this can be rewritten by

$$\left(\frac{\mu_r}{s}\right)\left(\frac{dH}{dt}\right)=\frac{\mu_r}{\psi}=H_i, \tag{6}$$

where H_i denotes the frequency-independent field. By means of (6), it is possible to represent (1) as

$$H+H_i=\left(\frac{1}{\mu}\right)B+\left(\frac{1}{s}\right)\left(\frac{dB}{dt}\right). \tag{7}$$

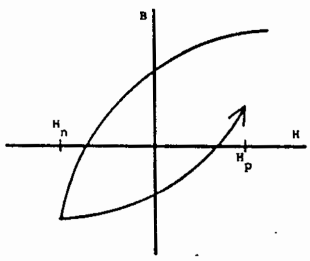

FIG. 1. Definitions of H_p and H_n.

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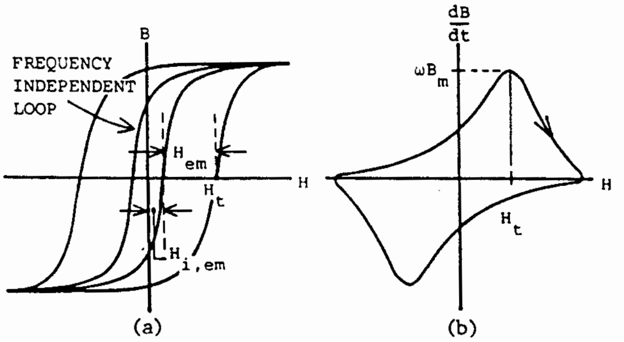

FIG. 2. Definitions of H_t, H_{em}, $H_{i,em}$, ωB_m.

B. Parameter *s*

When the flux density B is sinusoidary with time t, then the associated field H_t becomes a nonsinusoidal periodic wave. By means of a Fourier series, this field H_t can be classified into the odd H_{odd} and even H_{even} components,[11] that is,

$$H_t = H_{odd} + H_{even}. \tag{8}$$

Since the flux density B in (1) goes to zero when the time derivative of flux density dB/dt in (1) and the even component H_{even} in (8) arrive at their maximum values, (7) is reduced to

$$H_{even} = H_{em} + H_{i,em} = \left(\frac{1}{s}\right)\left(\frac{dB}{dt}\right), \tag{9}$$

where H_{em}, $H_{i,em}$ are shown in Fig. 2. By means of (9), the parameter s (called hysteresis coefficient) may be represented by

$$s = \left[\left(\frac{dB}{dt}\right) - \mu_r\left(\frac{dH_{odd}}{dt}\right)\right] \Big/ H_{em}$$

$$\simeq \frac{\omega B_m}{H_{em}} = f_s\left(\frac{dB}{dt}\right), \tag{10}$$

where $\omega = 2\pi f$ (f = frequency), $f_s(^\bullet)$ denotes a single valued function of $^\bullet$, and ωB_m is shown in Fig. 2.

C. Iron loss

By means of (7), iron loss per unit volume is given by

$$W = \frac{(\frac{1}{2})\,(\omega B_m)^2}{s} + f\oint H_{i,e}\, dB, \tag{11}$$

where $H_{i,e}$ is an even function of the frequency-independent field H_i of (6). Obviously, the first and second terms on the right of (11), respectively, correspond to the eddy current and hysteresis losses. Particularly, the eddy current loss W_e is related with parameter s as

$$W_e s = (\tfrac{1}{2})(\omega B_m)^2 \propto f^2 B_m^2. \tag{12}$$

Let us consider the barlike domain walls shown in Fig. 3, i.e., Sakaki[6] modified Pry and Bean's formula for eddy current loss W_e as follows:

$$W_e = 8.4 f^2 B_m^2 bd / \pi\rho n, \tag{13}$$

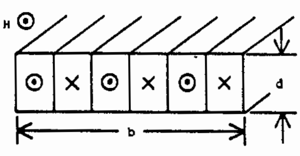

FIG. 3. Barlike domain walls model.

TABLE I. Constant of materials.

Marks	Materials	Thickness (mm)	Width (mm)	No. of layers
○	Grain-oriented 3Si97Fe	0.3	10	11
▽	Amorphous $Fe_{70}Ni_8Si_{10}B_{12}$	0.02	5	175
▲	Amorphous $Fe_4Co_{66}Si_{17}B_{13}$	0.02	10	174

where the sinusoidal flux density is assumed and n, ρ, d, b are, respectively, the number of barlike domain walls, resistivity, thickness, and width of material. Comparison of (12) with (13) yields the following relationship:

$$s = \pi^3 \rho n / 4.2bd. \tag{14}$$

D. Experiment

The composition and constant of the materials used for the experiment are shown in Table I. All samples are wound into a ring. Using a microcomputer, the iron loss measuring system calculates iron losses under sinusoidal flux conditions.[12]

Figure 4(a) shows the relationships of frequency, eddy current loss, and parameter s under a constant flux density. When we represent the curves by the functional relations shown in Fig. 4(a), then it is obvious that $u + v \simeq 2$ is established. This means that the dependency of $W_e s$ on frequency is proved by the results shown in Fig. 4(a). Similarly, Fig. 4(b) shows the relationships of flux density, eddy current loss, and parameter s under a constant frequency. As shown in Fig. 4(b), the curves can be represented by their functional relations. Consideration of these functional relations leads to $x + y \simeq 2$. The dependency of $W_e s$ on flux density is proven by the results shown in Fig. 4(b).

Furthermore, the functional relations of s in Figs. 4(a) and 4(b) reveal that the parameter s depends on the frequency as well as flux density. This proves the validity of Eq. (10). Also, it is obvious that the number of domain walls n in (14) is a function of the frequency and flux density as long as the resistivity ρ is held to constant value. According to (3) and (10), Preisach's function ψ is given by

$$\psi = s \Big/ \left(\frac{\partial H}{\partial t}\right) = f_s\left(\frac{dB}{dt}\right) \Big/ \left(\frac{\partial H}{\partial t}\right). \tag{15}$$

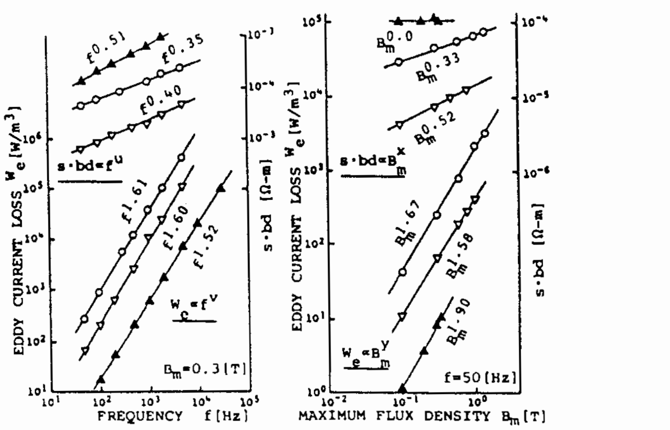

FIG. 4. (a) The relationships among the parameter s, eddy current loss, and frequency under a constant flux density. (b) The relationships among the parameter s, eddy current loss, and flux density under a constant frequency.

Obviously, the time derivatives of flux density and of the field are approximately proportional to the frequency, so that Preisach's function (15) may be nearly independent of frequency. Therefore, by considering this and (14), it may be considered that Preisach's function ψ represents the flux density dependency of the number of domain walls. Thus, the frequency-independent hysteresis loop may be reproduced by the Preisach-type model. Moreover, since the flux density B is proportional to the field intensity H in the weakly magnetized region, Preisach's function ψ becomes a constant. This weakly magnetized region and the constant may be called the Rayleigh's region and the Rayleigh's constant, respectively.

III. CONCLUSION

As shown above, we have discussed the various magnetization models, magnetization characteristics, and have clarified the relationships between them. As a result, a physical interpretation of the most important parameter in the Chua-type model has been successfully given by means of the barlike domain wall model.

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