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The locally orthogonal coordinate systems for inverse problem analysis

H. Saotome and Y. Saito
College of Engineering, Hosei University, 3-7-2 Kajino, Koganei, Tokyo 184, Japan

Abstract

Solving the inverse problem of magnetic field requires a large quantity of computation because solutions of this problem, i.e. current element (or dipole) vectors, are not only a function of their positions but also their directions. This paper proposes locally orthogonal coordinate systems with respect to magnetic field patterns on a measurement surface, such as the cylindrical and spherical coordinate systems, which can remove a repetitive calculation for determining the spatial angles of current dipole vectors. As a result, we have succeeded in realizing an efficient methodology for the inverse problems.

1. INTRODUCTION

Analyzing inverse problems is one of the most crucial future works or business in engineering because it can provide solutions for identification, optimization, automatic designing by computers, non-destructive testing, medical diagnosis and so forth. We have previously proposed a promising method called the sampled pattern matching (SPM) method for solving inverse problems in electromagnetic fields [1-6]. In the magnetostatic field, the inverse problem of obtaining current distribution from a locally measured magnetic field is carried out by estimating current element (or dipole) positions as well as the direction of each current dipole vector. Evaluating a spatial angle of each current dipole vector needs a repetitive process depending on the required angle resolution. For example, if five-degree resolution on a plane is required, then the SPM process of 72 times is repetitiously necessary at each of positions in order to cover the entire angle 360 degrees. In the previous studies, we have inevitably carried out the repetitive process with the Cartesian coordinate system [1-6]. However, this algorithm requires considerable CPU time.

The purpose of this paper is to find out the optimal coordinate systems that can remove the repetitive process for the current dipole angle evaluation. As a result, the locally orthogonal coordinate systems with respect to magnetic field components on a measurement surface, such as the cylindrical and spherical coordinate systems, are proposed in order to reduce the CPU time due to the current dipole angular division. This is because the magnetic field pattern on the measurement surface of these coordinate systems can be decomposed into two orthogonal field pattern components whereas it cannot be done in the Cartesian coordinate system.

2. INVERSE PROBLEMS OF HOMOGENEOUS MAGNETOSTATIC FIELD

2.1. Formulation

In the homogeneous open boundary magnetostatic field, the relation between the magnetic field intensity **H** and its source, the current density **J**, is expressed by a volume integral:

$$\mathbf{H} = \nabla \times \int_{V} G\, \mathbf{J}\, dv, \qquad (1)$$

where G is the Green function depending on the geometric relation between **H** and **J**.

In (1), the product J dv corresponds to the current dipole vector.

Employing the cylindrical coordinate system as shown in Figure 1, the magnetic field intensity H_z normal to the measurement surface can be decomposed into two components H_{zr} and $H_{z\theta}$:

$$H_z = H_{zr} + H_{z\theta}, \tag{2}$$

where H_{zr} and $H_{z\theta}$ are caused by r and θ components of the current dipole vector $\boldsymbol{\alpha}$ (= J dv) shown in Fig. 1, respectively. With unit vectors of the cylindrical co-ordinates, $\mathbf{e}_r$, $\mathbf{e}_\theta$ and $\mathbf{e}_z$, we have

$$H_{zr} = \frac{(\alpha_r\ \mathbf{e}_r)\times\mathbf{s}}{4\pi s^3}\cdot\mathbf{e}_z = \frac{\alpha_r\ \xi}{4\pi s^3}, \tag{3a}$$

$$H_{z\theta} = \frac{(\alpha_\theta\ \mathbf{e}_\theta)\times\mathbf{s}}{4\pi s^3}\cdot\mathbf{e}_z = \frac{\alpha_\theta\ \eta}{4\pi s^3}, \tag{3b}$$

$$\alpha_r = \boldsymbol{\alpha}\cdot\mathbf{e}_r, \tag{3c}$$

$$\alpha_\theta = \boldsymbol{\alpha}\cdot\mathbf{e}_\theta, \tag{3d}$$

$$s = |\mathbf{s}|, \tag{3e}$$

where the distance vector $\mathbf{s}$, ξ and η are shown in Fig. 1. Moreover, we obtain

$$\int_P H_{zr}\ H_{z\theta}\ dS = 0, \tag{4}$$

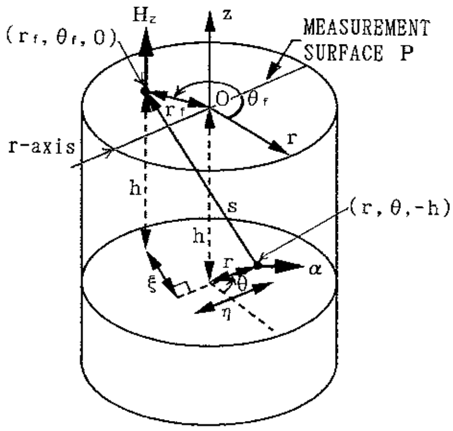


Figure 1. Cylindrical coordinate system.

where P refers to a circular measurement surface area shown in Figure 1. This is caused by the fact that the magnetic field patterns composed of H_{zr} and $H_{z\theta}$ in Figure 1 are odd and even symmetrical functions with respect to the r-axis, respectively.

Similarly, the following formulation can be carried out with the spherical coordinate system shown in Figure 2. The magnetic field intensity normal to the spherical measurement surface in Figure 2, H_r, can be represented by

$$H_r = H_{r\theta} + H_{r\phi}, \tag{5}$$

where $H_{r\theta}$ and $H_{r\phi}$ are caused by θ and ϕ components of the current dipole vector $\boldsymbol{\alpha}$ shown in Figure 2, respectively. Using the distance vector $\mathbf{s}$, unit vectors $\mathbf{e}_r$, $\mathbf{e}_\theta$ and $\mathbf{e}_\phi$ in the spherical coordinate system shown in Figure 2, we get

$$H_{r\theta} = \frac{(\alpha_\theta\ \mathbf{e}_\theta)\times\mathbf{s}}{4\pi s^3}\cdot\mathbf{e}_r, \tag{6a}$$

$$H_{r\phi} = \frac{(\alpha_\phi\ \mathbf{e}_\phi)\times\mathbf{s}}{4\pi s^3}\cdot\mathbf{e}_r, \tag{6b}$$

$$\alpha_\theta = \boldsymbol{\alpha}\cdot\mathbf{e}_\theta, \tag{6c}$$

$$\alpha_\phi = \boldsymbol{\alpha}\cdot\mathbf{e}_\phi, \tag{6d}$$

and

$$\int_P H_{r\theta}\ H_{r\phi}\ dS = 0, \tag{7}$$

where P refers to a spherical measurement surface area shown in Figure 2.

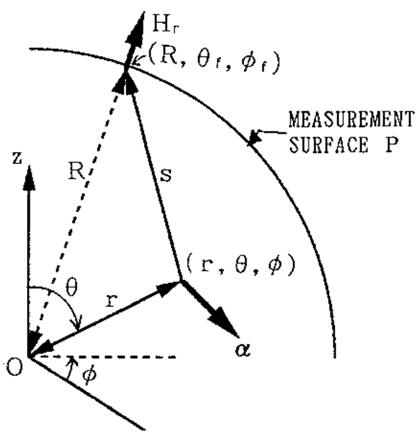


Figure 2. Spherical coordinate system.

Discretizing (1) into m volume subdivisions gives a system equation of the inverse problem:

$$\mathbf{u} = \sum_{j=1}^{m} (\alpha_{pj}\, \mathbf{d}_{pj} + \alpha_{qj}\, \mathbf{d}_{qj}); \tag{8}$$

p=r, q=θ for the cylindrical coordinate system;
p=θ, q=Φ for the spherical coordinate system;

where the elements of **u** are measured magnetic field intensities normal to the measurement surface, α_{pj} and α_{qj} are p and q components of a current dipole vector at the point j. The vectors $\mathbf{d}_{pj}$ and $\mathbf{d}_{qj}$ show the estimated magnetic field patterns on the measurement surface due to α_{pj} and α_{qj}, respectively.

2.2. Faster SPM method

The most notable point of our new formulation is the orthogonality between two magnetic field component patterns expressed by (4) and (7). This orthogonality is not satisfied in the commonly used Cartesian coordinate system. Equations (4) and (7) give

$$\mathbf{d}_{pj}{}^{T} \cdot \mathbf{d}_{qj} = 0; \quad j=1,2,\cdots,m. \tag{9}$$

This relation makes it possible to independently evaluate the estimated magnetic field pattern matching rates of p and q component patterns with respect to the measured magnetic field pattern. Namely,

$$\gamma_{pj} = \mathbf{u}^{T} \cdot \mathbf{d}_{pj} \;/\; (\|\mathbf{u}\|\,\|\mathbf{d}_{pj}\|), \tag{10a}$$

$$\gamma_{qj} = \mathbf{u}^{T} \cdot \mathbf{d}_{qj} \;/\; (\|\mathbf{u}\|\,\|\mathbf{d}_{qj}\|), \tag{10b}$$

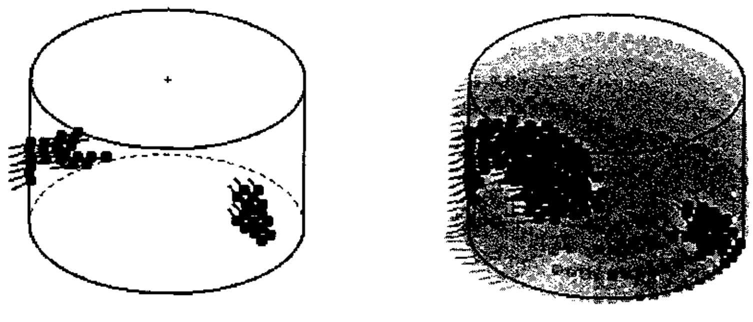
where j=1,2,$\cdots$,m. Therefore, current dipole searching is carried out by finding out the maximum of

$$\gamma_{j} = \sqrt{\gamma_{pj}{}^{2} + \gamma_{qj}{}^{2}}; \quad j=1,2,\cdots,m. \tag{10c}$$

After finding out the first current dipole vector, we continue with the procedures similar to (10a)-(10c) up to the peak of γ [1-6]. Making the most of (9), the SPM algorithm is carried out only in p and q directions instead of taking all the divided spatial angles of a current dipole into account.

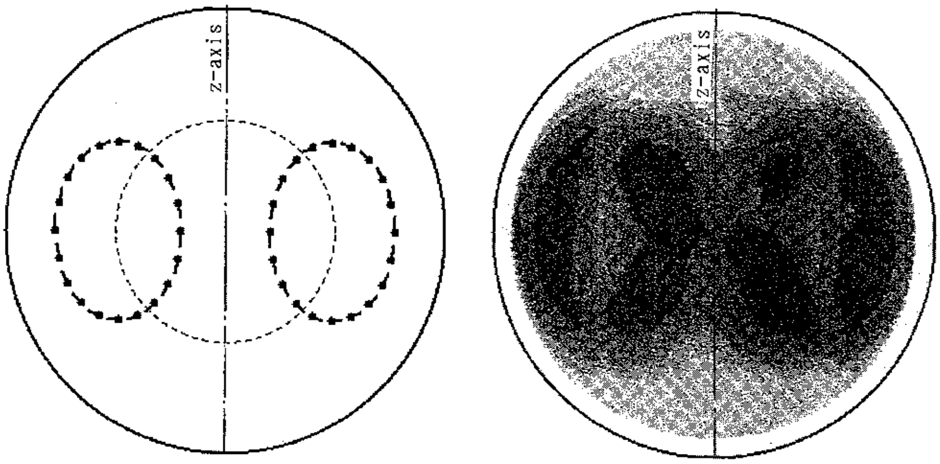
3. Examples

Applying the proposed formulation, current dipole distribution is quickly



(a) (b)

Figure 3. Current dipole distributions in the cylindrical coordinate system.
(a)Correct distribution; (b)estimated distribution (n=37, m=2046).



(a) (b)

Figure 4. Current dipole distributions in the spherical coordinate system. (a)Correct distribution; (b)estimated distribution (n=407, m=105245).

obtained without the CPU time required to determine each angle of current dipoles. Figure 3 shows correct and estimated current dipole distributions obtained by employing the new method with the cylindrical coordinate system. Figure 4 shows an example of correct and estimated current distributions in the spherical coordinate system. The field source searching region of this spherical example ranges in radial direction from 0.5 to 0.9 regarding a radius of the sphere as one. The number of measurement points, n, and the number of unknowns, m, are n=37, m=2046 in Fig. 3 and n=407, m=105245 in Fig. 4, respectively.

4. CONCLUSION

As shown above, we have formulated the inverse problem of magnetostatic field by means of the locally orthogonal coordinate systems. This enables us to obtain current dipole distribution faster than the conventional algorithm employing the Cartesian coordinate system. The novel algorithm proposed in this paper leads to fast medical diagnosis with a magnetocardiogram or a magnetoencephalogram.

5. REFERENCES

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