IMAGE PROCESSING BY FIELD THEORY

–PART 2 : APPLICATIONS-

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Abstract. This paper proposes one of the image analysis methodologies for visualizing the electromagnetic fields. A key idea is that each of the pixels representing a digital image is regarded as a kind of potentials in vector fields. The vector calculus to the image yields the image differential equations. The Poisson and Helmholtz types of equations are possible to represent the static and dynamic images, respectively. In this paper, visualized electromagnetic field images are analyzed based on our image differential equations. The image Poisson equation is possible to reconstruct the original electromagnetic fields with high spatial resolution. The image Helmholtz equation enables us extraction of the parameters characterizing the electromagnetic phenomena.

Keywords: Image Helmholtz equation, Image Poisson equation, Magnetic field, Magnetic domain, Visualization

INTRODUCTION

Visualization of electromagnetic fields is most important interface for designing electric and electronic devices, non-destructive evaluations, electromagnetic compatibility, education, and so on. With the various numerical approaches, the electromagnetic field distribution is estimated by solving Maxwell's equations. This has spurred developing high intelligent tools and visualizing the electromagnetic phenomena. At the same time, the development of various visualizing devices gives reliable sources of information on natural phenomena. Consideration of visualized images facilitates clarification physics of the complex phenomena. Furthermore, modern personal computers make it possible to process enormous data obtained by various ways. In case of the visualized images, the computers are capable of contributing to understanding of more complex phenomena. Therefore, it is essential to develop visualized image analysis methodologies, which enable to extract and/or to visualize the characteristics involving the images.

Principal purpose of this paper is to apply the classical field theory to the digital image analysis. A key idea is that each of pixels constituting a digital image is regarded to be a kind of potential in vector fields. Based on this assumption and the vector calculus, the image governing equations can be derived. The Poisson and Helmholtz types of equations can represent the static and dynamic images, respectively [1], [2]. The image differential equations are applied to visualized electromagnetic images in this paper. At first, solving the image Poisson equation generates the magnetic field distribution with arbitrary resolution. Secondly, magnetic domain dynamics is evaluated by characteristic value derived from the solution of the image Helmholtz equation employing modal analysis.

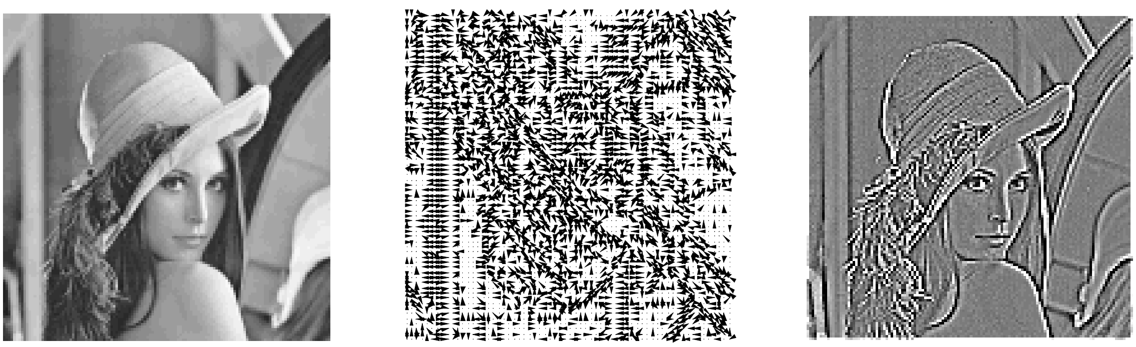
VISUALIZATION BY IMAGE POISSON EQUATION

Vector Field of Digital Images and Image Poisson Equation

Our image analysis methodology is an application of the classical field theory. First of all, the key idea is described. The image data contained in the pixels could be regarded as a scalar potential or one of the components of vector potential. When we denote U as scalar potential, gradient operation to the potential yields the gradient fields. For example, gradient operation to an image in Fig. 1(a) yields the gradient vectors as shown in Fig. 1(b). Moreover, divergence operation to the gradient vector field shown in Fig. 1(b) yields the source density field as shown in Fig. 1(c). As a result of these operations to an image, the governing equation of a static image can be derived. Namely, if we can recover the original image from the source density field shown in Fig. 1(c), the Poisson equation is satisfied as governing equation [1].

(1) $$-\nabla^2 U = \sigma,$$

where σ represents the source density field derived by the Laplacian of the image field U. By solving for Eqn. 1 using Fig. 1(c) as source density field σ, it is possible to recover the image as shown in Fig. 2(a). Calculation of Eqn. 1 is carried out by the finite difference method in this paper. The correlation analysis between Figs. 1(a) and 2(a) becomes 1. Therefore, it is confirmed that the Poisson equation is possible to represent the digital images. For the nature of differential equation, higher resolution image of Fig. 1(a) can be generated. Employing fine mesh system of the finite difference method makes it possible to generate the higher resolution image as shown in Fig. 2(b). The image representation by differential equation leads to the generalization of digital images.



(a) Original image (b) Gradient operation to Fig. 1(a) (c) Source density field

Figure 1. Vector field representation of a digital image.

(a) Recovered from 128x128 to 128x128 pixels

(b) Generated from 128x128 to 256x256 pixels

Figure 2. Image recovery from the source density field by means of the Poisson equation.

Magnetic Field Imaging by the Image Poisson Equation

All of the color images are composed of the three primary colors, i.e., red, green, and blue. Suppose that one color component of a digital image is composed of scalar field *U*. Then the color image can be analyzed by three Poisson equations.

(2) $$-\nabla^2 U_{color} = \sigma_{color}, \quad color = Red, Green, Blue,$$

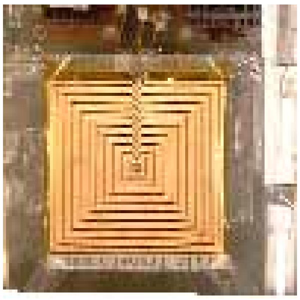
The solution of Eqn. 2 is capable of representing the image with any resolution. As an application of the image Poisson equation, spatial resolution of magnetic field distribution over a DC/DC converter shown in Fig. 3(a) is improved. At first, *x*, *y*, and *z* components of measured magnetic field as shown in Fig. 3(b) are projected onto red, green, and blue components of a color image, respectively. Then, visualized magnetic field can be obtained as shown in Fig.3(c). Solving Eqn. 2 in terms of each color component gives the magnetic field distribution with higher spatial resolution as shown in Fig. 3(d). In general, the spatial resolution improvement of magnetic as well as electric fields could be accomplished by employing the inverse solution approach which evaluates the current or charge distribution from the measured fields. In that case, it is impossible to avoid solving for the ill-posed linear system of equation. However, our field visualization by the Poisson equation makes it possible to realize with simple image handling technique.

VISUALIZATION BY IMAGE HELMHOLTZ EQUATION

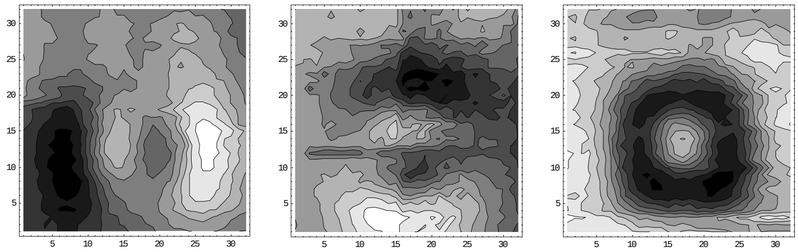
Image Helmholtz Equation

To represent the visualized physical dynamics, we derived a Helmholtz equation as a governing equation, since one frame of the dynamic image is governed by the Poisson equation. The key idea is that Helmholtz types of equations govern many of the physical dynamic systems. Assuming a frame to be described in terms of a scalar field *U*, any dynamic image can be given as a solution of the Helmholtz equation [2]:

(3) $$\nabla^2 U + \varepsilon \frac{\partial}{\partial \alpha} U = -\sigma,$$

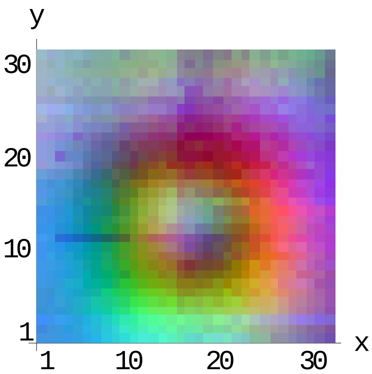


(a) DC/DC converter utilizing a film-type transformer

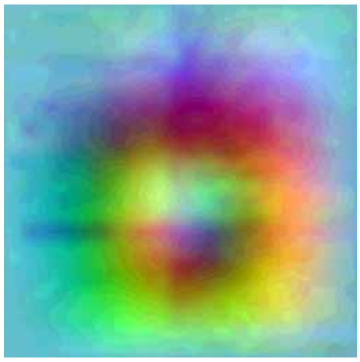


x component y component z component

(b) Measured magnetic field distribution (32x32 points, 5 mm/point)



(c) Visualized magnetic field (32x32 pixels, 5 mm/pixels)



(d) High-spatial resolution magnetic field reconstructed from Fig. 3(c) (256x256 pixels, 0.63 mm/pixel)

Figure 3. Spatial resolution improvement of measured magnetic field distribution around a DC/DC converter

where ε and α denote the moving speed parameter and transition variable, respectively. The first and second terms on the left in Eqn. 3 represent the spatial expanse and transition of frame to the variable α, respectively. The parameter ε in Eqn. 3 is not given. However, this means that it is possible to extract the characteristic of physical properties from the visualized dynamic image if it is possible to determine the ε from given frames of the dynamic image.

Let us consider the modal analysis to Eqn. 3. Then the general solution can be derived as

(4) $$U(\alpha) = \exp(-\Lambda\alpha)\left(U_{Start} - U_{Final}\right) + U_{Final},$$

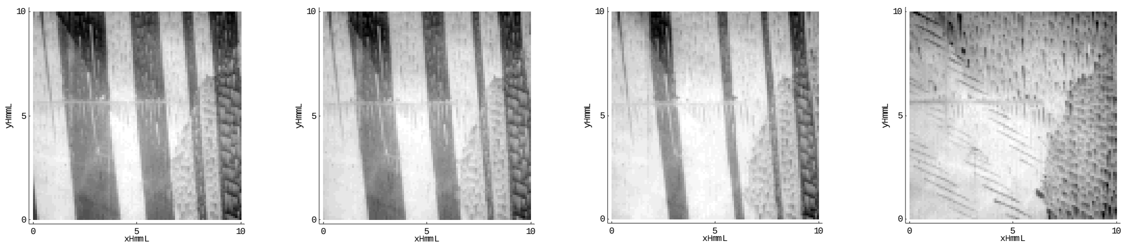
where U_{Start} and U_{Final} represent an initial and a final frame of the dynamic image, respectively. Moreover, exp(-$\Lambda\alpha$) denotes the state transition matrix. The values ε and σ in Eqn. 3 are respectively reduced into the matrix Λ and final image. Because of the parameter ε in Eqn. 3, the state transition matrix is unknown. However, if the solution $U(\alpha)$ is assumed to be one of the frames of dynamic image, it is possible to determine the elements in matrix Λ, as given by

(5) $$\Lambda = -\frac{1}{\alpha}\ln\left(\frac{U(\alpha) - U_{Final}}{U_{Start} - U_{Final}}\right).$$

When we solve the differential equations, all of medium parameters are given. However, in this image analysis, determination of the matrix Λ by means of Eqn. 5 is capable of extracting the characteristics of visualized physical dynamics. This is a distinguished feature of our image analysis methodology. As a result, substituting the matrix Λ into Eqn. 4 yields dynamic image generation at the arbitrary α.

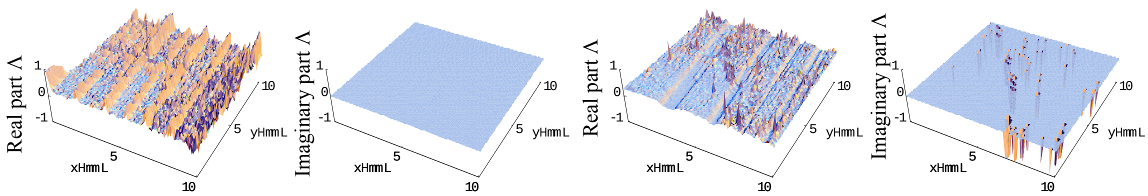
Visualization of Magnetic Domain Behavior by the Image Helmholtz Equation

When dynamic image of a grain-oriented silicon steel sheet is visualized, it is possible to extract the physical parameters of magnetization by means of the image Helmholtz equation. In this case, the transition variable α is assumed to be the applied field. Fig.4 shows the distinct magnetized domain images observed by scanning electron microscope (SEM). Fig.5 illustrates the matrices Λ at each of the magnetized regions. Since the real and imaginary parts of matrix Λ in Eqn. 4 respectively represent in phase and 90-degree phase difference components to the applied field, then the domain motion speed and iron loss distribution can be visualized as the real and imaginary parts in Fig.5, respectively.



(a)H=0.0 A/m, B=0.0 T (b)H=2.9 A/m, B=0.1 T (c)H=9.3 A/m, B=1.6 T (d)H=115.4 A/m, B=1.88 T

Figure 4. Magnetic domain SEM images of the ORIENTCORE HI-B produced by Nippon Steel Corporation. (100x100 pixels, 0.1 mm/pixel)

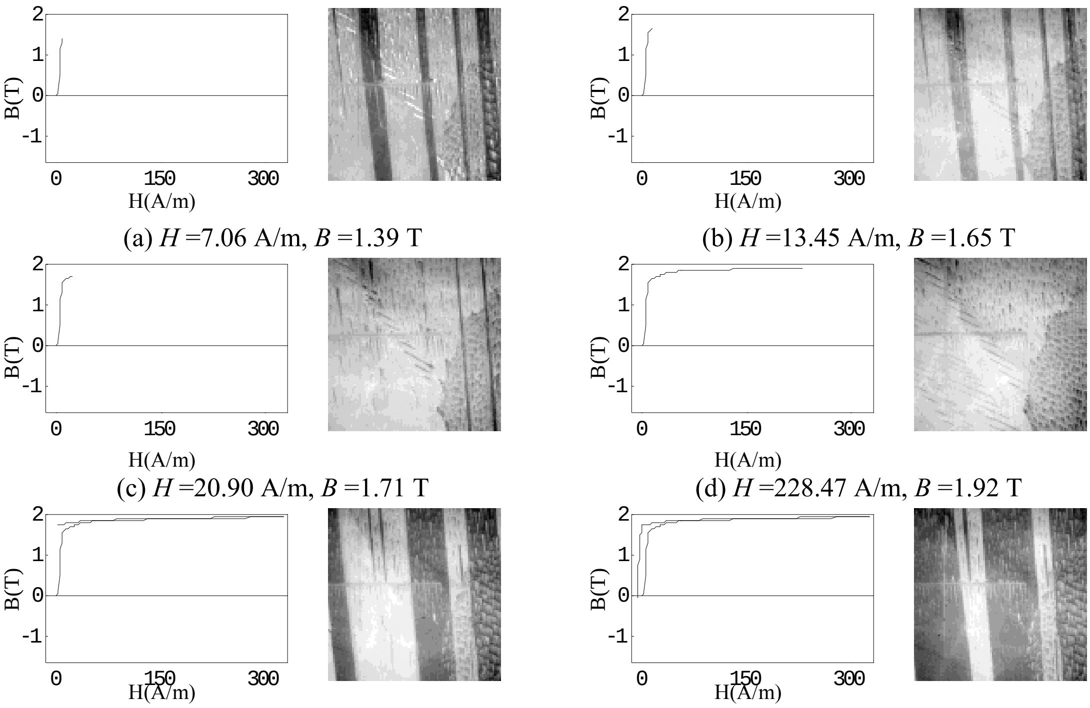


Real Part Imaginary Part Real Part Imaginary Part

(a) $0.0 \le H < 2.9$ A/m (b) $2.9 \le H < 9.3$ A/m

Figure 5. State transition matrices.

The contrast of SEM images shown in Fig.4 corresponds to the polarity of magnetization. A summation of all magnetization gives a flux density in a magnetic material. Computing an average of pixel value on entire domain image just corresponds to a flux density in a magnetic material. Fig.6 shows the computed magnetization curves together with generated domain images. Since Eqn. 4 is capable of generating the domain images analytically, then the smooth computed magnetization curve is generated.



(a) H =7.06 A/m, B =1.39 T

(b) H =13.45 A/m, B =1.65 T

(c) H =20.90 A/m, B =1.71 T

(d) H =228.47 A/m, B =1.92 T

(e) H =2.32 A/m, B =1.76 T

(f) H=-6.39 A/m, B =-0.06 T

Figure 7. Reconstructed domain images and magnetization curves.

CONCLUSIONS

This paper has introduced the visualization and analyzing methodologies based on the classical field theory along with practical examples. The Poisson and Helmholtz types of equations can represent the static and dynamic images, respectively. In an application of the image Poisson equation to the magnetic field imaging, the spatial resolution improvement has been carried out without solving for the ill-posed linear system of equations. In an application of the image Helmholtz equation to the magnetic domain images, visualization of the magnetic motion dynamics has been worked out by considering the matrices Λ derived from the SEM domain images. Thus, it is obvious that our approach has an extremely high possibility to extracting the physical properties from any visualized images.

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