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Modal-Wavelet Transform for Smart Visualization Tool

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Abstract: This paper proposes the modal-wavelet transform (MWT) as one of the orthogonal wavelet transforms. The theoretical background and application of our MWT are described. The bases of MWT are derived from modal analysis of the potential field equations. Namely, principal idea of MWT is that a numerical data set is regarded as a set of field potentials or source densities. A modal matrix, constituting characteristic vectors, derived from the discretized field equations enables us to carry out an orthogonal transform in much the same way to those of the conventional discrete wavelets. The modal-wavelets based on the data modeling provide a multi-resolution analysis in most efficient manner. Applying 3-dimensional analysis of the MWT to a weather satellite infrared animation classifies it into background and cloud moving frame images.

Keywords: Modal-Wavelets, Discrete Wavelets

1. Introduction

The spread of high performance and reasonable price computers has yielded a large scale Internet community as well as information resources. Data handling technologies based on the digital computers are of main importance to realize more efficient networking and computing. Discrete wavelet transform (DWT) may be promised to become a deterministic methodology handling the digital signals and images, e.g., compressing data quantity, extracting their characteristics, etc (e.g., Matsuyama, 1999). Moreover, their applications to electromagnetic field calculation, solving forward and inverse problems, have been investigated and spurred to faster calculation algorithm (Beylkin et al., 1991, Doi et al., 1996). The conventional DWT, however, sometimes suffers from limitation on subject data length, which must be power of 2. Thereby, the applications depend on employed wavelet basis, and it needs an enormous memory installation for implementation. The principal purpose of this paper is to derive new wavelet basis to carry out more efficient wavelet analysis.

This paper proposes the MWT as one of the DWTs. The bases of MWT are derived from a modal analysis of the discretized field equations. Regarding a numerical data set as the potential or source density distribution leads to a discretized data model, i.e., the data set can be represented by the field equation, e.g., Poisson's equation. Then, the modal analysis of the discretized field equation gives a modal matrix constituting characteristic vectors. The modal matrix enables us orthogonal transform in the same nature as DWT. MWT utilizes this matrix as one of the wavelet bases. Because of the field equation based modeling, MWT makes is possible to generate an optimal basis to the subject data length.

2. Modal-Wavelet Transform

2.1 Data Representation by Field Theory

To derive a new wavelet bases, we consider a discrete data modeling based on the classical field theory. Namely, a numerical data set is assumed to be the potential or source fields. According to the field theory a scalar field u caused by source density σ could be obtained solving the differential equation, i.e., Poisson equation:

$$\varepsilon \nabla^2 u = -\sigma \tag{1}$$

where ε is the medium parameter of the field. Also, the scalar field u can be obtained by fundamental solution

$$u = \frac{1}{\varepsilon} \int g(r)\sigma \quad dr \tag{2}$$

where $g(r)$ is a Green function; r is the distance from the source to reference points.

Discretization of (1) and (2) by numerical methods respectively derive the following system of equations:

$$L\mathbf{U} = \mathbf{f} \tag{3}$$

and

$$G\mathbf{f} = \mathbf{U} \tag{4}$$

where $\mathbf{f}$, $\mathbf{U}$ denote the vectors corresponding to the source density σ and the scalar field u; and L, G denote the coefficient matrices derived from the Laplacian operator in (1) and Green function in (2), respectively.

As an example, let each of pixel values in Fig.1(a) be a scalar potential assuming the medium parameter ε to be a constant on the entire field, then applying L or G^{-1} to Fig.1(a) yields the source density distribution like Fig.1(b). Solving (3) or (4) with the source density as vector $\mathbf{f}$ reproduces the image as in Fig.2. Especially, Figs. 1(a) and 2(a) are identical in values. Therefore, our discrete data modeling based on field equation is capable of representing numerical data sets (Endo et al., 2001).

(a) Original Image (128x128 pixels)

(b) An Example of Source density (128x128 pixels)

Fig. 1 Source Density Representation of a 2D Image

(a) Recovered Image by Poisson Equation (3)

(b) Recovered Image by Integral Equation (4)

Fig. 2 Image Recovery from Image Source Density (128x128 pixels)

2.2 Modal-Wavelet Transform

As is well known, the matrices L in (3) and G in (4) derived by available discretizing methods, e.g., finite elements etc., become the symmetrical as well as positive definite matrices. In case when the vector $\mathbf{U}$ has q elements, it is possible to obtain the characteristic values λ_i, i=1,2,…,q, of the matrices L and G and their respective characteristic vectors $\mathbf{v}_i$, i=1,2,…,q. The matrix composed of the characteristic vectors $\mathbf{v}_i$, i=1,2,…,q as its columns is called the modal matrix:

$$M_q = \left(\mathbf{v}_1 \quad \mathbf{v}_2 \quad \dots \quad \mathbf{v}_q\right) \tag{5}$$

Because of the orthogonality, it holds following relationship:

$$M_q^{\,T} M_q = I_q \tag{6}$$

where the superscript T refers to a matrix transpose and I_q is a q by q unit matrix. The modal matrix derived from the coefficient matrix L or G has the same nature as those of the conventional DWT matrices. Moreover, a linear combination of the characteristic vectors is possible to represent the value distribution in a data set. Thus, MWT employs this modal matrix as DWT matrices.

2.3 Modal-Wavelet Transform Matrix and Basis

The MWT matrices can be derived various methods of discretizations. The MWT matrices introduced in the present paper are classified into two types. One is differential equation type assumed the subject data to be a potential field. The other is integral expression type assumed the subject data to be the field source distribution.

At first, let us consider MWT derived from differential equation. The simplest system matrix L can be obtained by one-dimensional Laplacian operation with equi-meshed 3 points finite difference approximation. Namely, the matrix L in (3) is given by

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} \cong U_{x-1} - 2U_x + U_{x+1}, \qquad x = 1,2,\dots,q \tag{7}$$

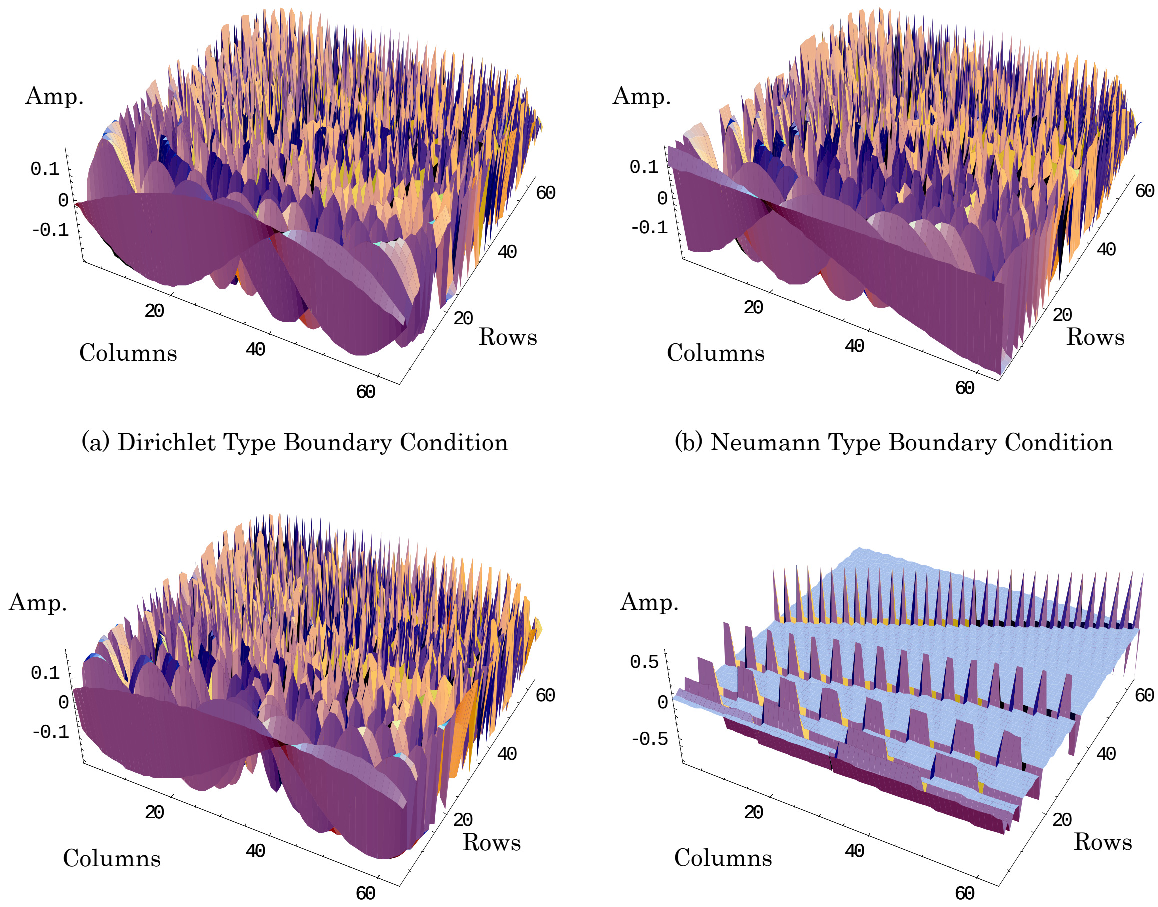
Then, applying the Jacobi method yields a modal matrix M_q in (5). Therefore, the dimension of matrix M_q depends on number of subdivision of (7). This means it is possible to generate an optimal basis to the subject data. In the Laplace partial differential equation, two types of boundary conditions should be considered, i.e., the Dirichlet- and Neumann- type boundary conditions. Figs.3(a) and (b) illustrate the typical differential equation based MWT matrices. As shown in

Figs.4 and 5, the bases having the Dirichlet- and Neumann- type boundary conditions become odd- and even- functions, respectively. The bases of MWT look like sinusoidal functions, however, the bases are not composed of the single frequency component. Moreover, the elements constituting the transform matrices never become the complex numbers like the Fourier transform.

Second, let us consider MWT derived from integral expression. We consider a 3-dimensional Green function $g(r)$ in (2). However, the 3-dimensional Green function takes infinity when $g(0)$ due to integral kernel. In order to remove this difficulty the matrix G in (4) is given by assuming the minimum distance $r_{i,i} = 1$, thus,

$$g(r) \cong \begin{cases} \dfrac{1}{r_{i,j}} & i \neq j \\ & \text{if} \quad , \qquad i = 1,2,\ldots,q, \qquad j = 1,2,\ldots,q \\ 1 & i = j \end{cases} \tag{8}$$

where the subscripts i and j refer the source and reference points, respectively. Thereby, $r_{i,j}$ represents the distance between them. Since the system matrix derived from (8) becomes symmetrical, then the Jacobi method can be applied to obtain its modal matrix in much the same way as the MWT based on differential equation. Figs. 3(c) and 6 show the MWT matrix and its bases. They have the similar patterns to that of the MWT matrix derived under the Dirichlet boundary condition.



(a) Dirichlet Type Boundary Condition

(b) Neumann Type Boundary Condition

(c) Green Function Type

(d) Daubechies 2nd Order

Fig. 3 Modal-Wavelet matrices (64 x 64)

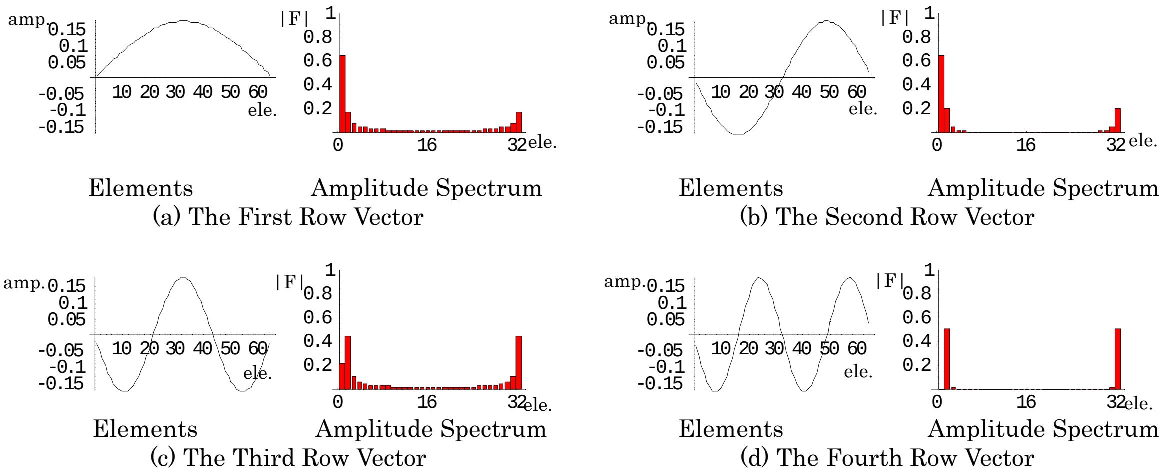


Fig. 4 Elements of Row Vectors in the matrix shown in Fig.3(a) and Their Fourier amplitude spectrum

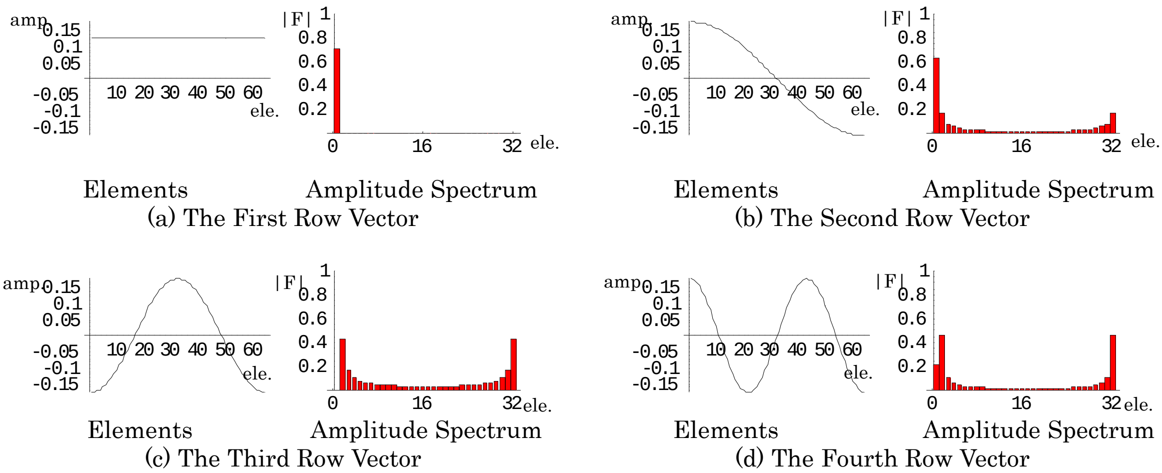


Fig. 5 Elements of Row Vectors in the matrix shown in Fig.3(b) and Their Fourier amplitude spectrum

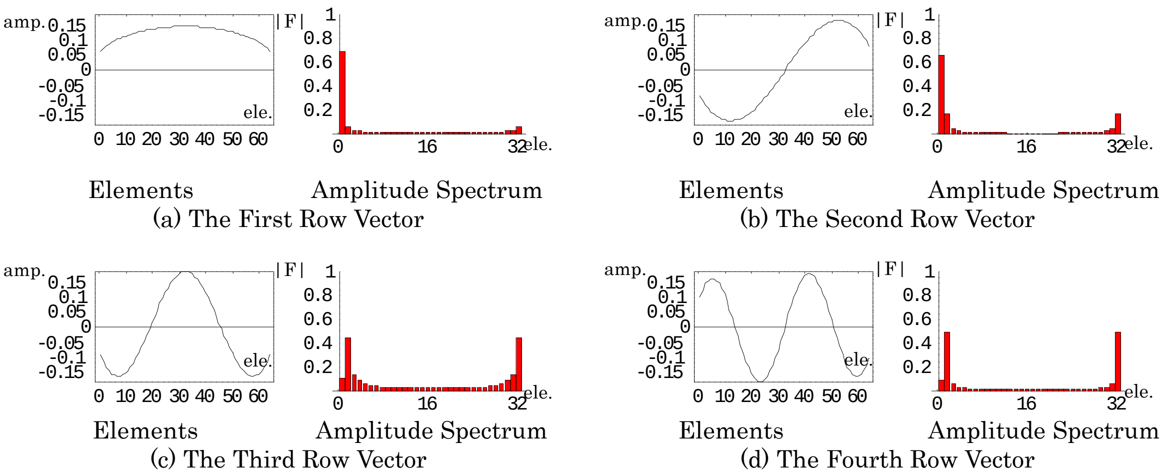
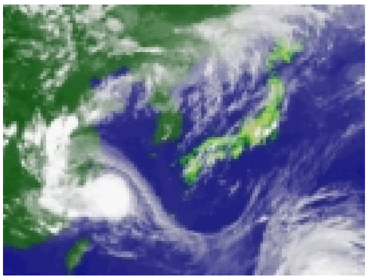


Fig. 6 Elements of Row Vectors in the matrix shown in Fig.3(c) and Their Fourier amplitude spectrum

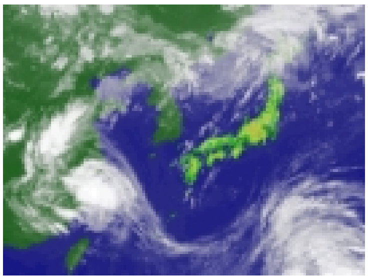
3. Application to Animation Image Analysis

3.1 Infrared Animation of Weather Satellite

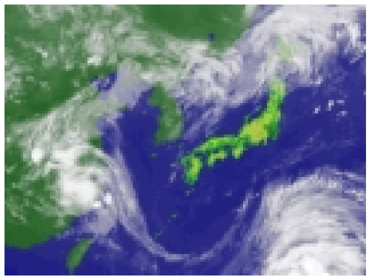
Fig.7 shows some frames of an infrared animation observed by the weather satellite Himawari (http://www.jwa.or.jp/). Applying MWT to this animation, separation of static and dynamic images is demonstrated. The animation used in this example is composed of 22 frames captured from 18:00 Aug. 10th to 15:00 Aug. 11th in 2000. Fig.7 shows the generation process of typhoon No. 9 in 2000.



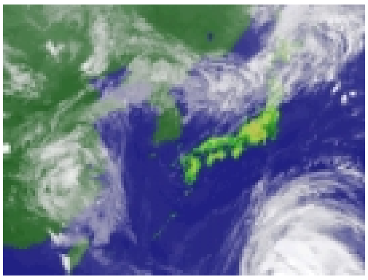
(a) At 18:00, Aug. 10, 2000



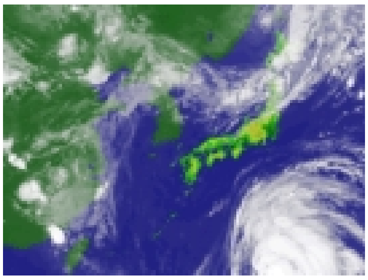
(b) At 22:00, Aug. 10, 2000



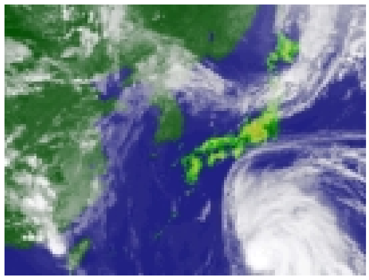
(c) At 2:00, Aug. 11, 2000



(d) At 6:00, Aug. 11, 2000



(e) At 10:00, Aug. 11, 2000



(f) At 14:00, Aug. 11, 2000

Fig. 7 Frames of Infrared Animation by Weather Satellite Himawari (256x193 pixels)

3.2 3-Dimensional Modal-Wavelet Transform

In order to apply MWT to the animation in Fig.7, the 3-dimensional MWT is essential. Namely, applying MWT to horizontal-, vertical- and frame- axes of the animation carries out animation analysis. Let us consider the animation S_{lmn} having m x n pixels and l frames. Then, its transpose rules are defined by

$$[S_{lmn}]^T = S_{mnl},\ [S_{mnl}]^T = S_{nlm},\ [S_{nlm}]^T = S_{lmn} \tag{9}$$

The 3-dimensional MWT gives the modal-wavelet spectrum S_{lmn}':

$$S_{lmn}' = \left[M_n\left[M_m[M_l S_{lmn}]^T\right]^T\right]^T \tag{10}$$

where M_l, M_m and M_n are the l by l-, m by m- and n by n- MWT matrices, respectively. And then, inverse MWT recovers the original animation S_{lmn}:

$$S_{lmn} = M_l^T\left[M_m^T\left[M_n^T\left[S_{lmn}'\right]^T\right]^T\right]^T. \tag{11}$$

Since a linear combination of weighted spectrum represents the original animation S_{lmn}, therefore, animation of each level can be generated by means of (11).

3.3 Separation of Static and Dynamic Images

As shown in Figs.3(b) and 5(a), the lowest level of bases derived under the Neumann boundary condition is a constant term. This means that the multi-resolution analysis to the frame axis is capable of extracting a common static image through entire frames of animation when employing the Neumann type MWT matrix. In much the same way, the dynamic frame images of animation can be extracted.

Figs.8 and 9 show the results of the multi-resolution analysis to the frame axis. Taking the lowest level of MWT multi-resolution analysis (11) into account yields the image in Fig.8. In this case, the generated result has some frames, but all of frames are the same as Fig.8. Thus, Fig.8 is the extracted background image representing static air pressure distribution. On the other hand, Fig.9 shows dynamic frame images of animation obtained by means of (11) without the lowest level of spectrum.

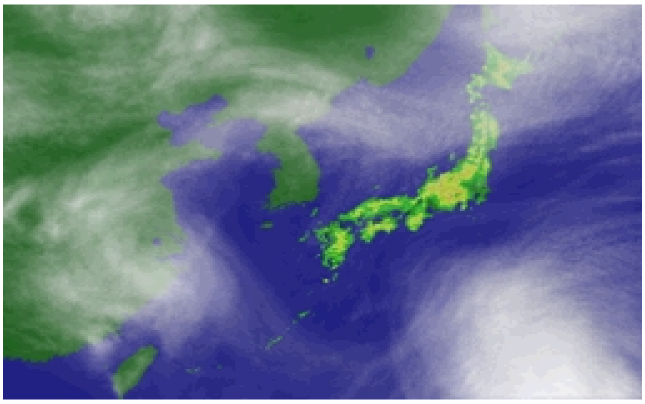
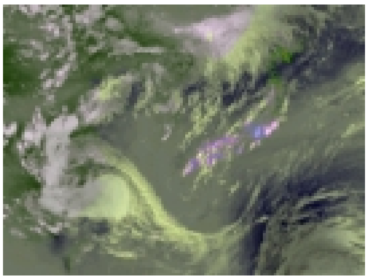
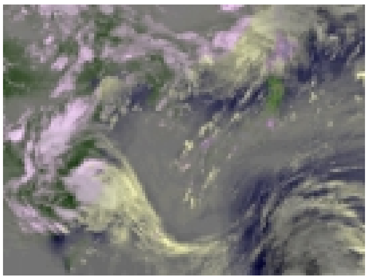


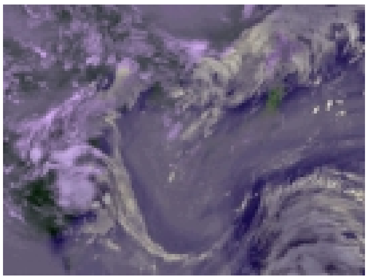
Fig. 8 Extracted Static Image (256 x 193 pixels)



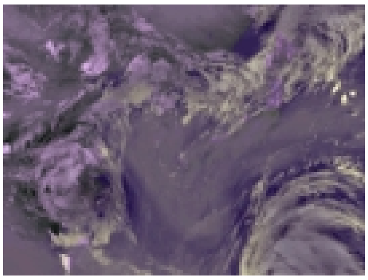
(a) At 18:00, Aug. 10, 2000



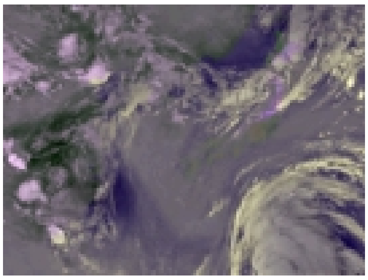
(b) At 22:00, Aug. 10, 2000



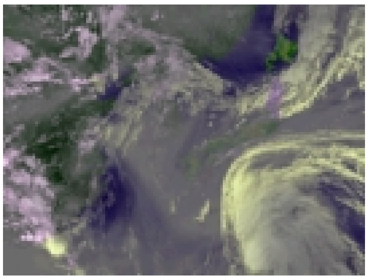
(c) At 2:00, Aug. 11, 2000



(d) At 6:00, Aug. 11, 2000



(e) At 10:00, Aug. 11, 2000



(f) At 14:00, Aug. 11, 2000

Fig. 9 Frames of Extracted Dynamic Image (256 x 193 pixels)

3.4 Comparison with Conventional Wavelets

In the conventional DWT, the data length l, m and n must be a power of 2. In this animation analysis, the animation shown in Fig.7 has 256 x 193 pixels and 22 frames. If we carry out the same analysis with conventional DWT, then l, m and n described in *Section 3.2* become 32, 256 and 256, respectively. In case of MWT, l, m and n are 22, 256 and 193, respectively. MWT accomplishes an efficient analysis from the viewpoint of memory consumption.

4. Conclusion

We have proposed the MWT and shown its application to animation analysis. Data representation by field equations has provided the optimal MWT bases to subject data. Application to animation analysis has demonstrated the separation of static and dynamic images with high efficiency compared with those of conventional DWT. As shown above, our MWT approach has versatile capability not only to information resource handling but also smart computing.

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